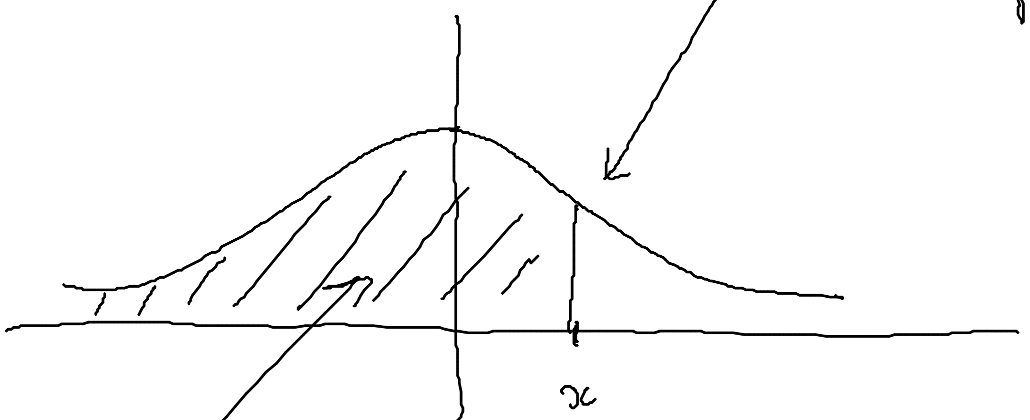
$$X \sim N(1,4)$$

$$f(x) = \frac{1}{\sqrt{2\pi}\cdot 2} e^{-\frac{(x-1)^2}{8}}$$

$$P(X \le 2{,}6)$$

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \longleftarrow N(0,1)$$



$\Phi(x)$

Se $X \sim N(\mu, \sigma^2) \Rightarrow Y = \frac{X-\mu}{\sigma} \sim N(0, 1)$

Se X ha speranza μ e varianza σ^2

$Y = \underbrace{\frac{X-\mu}{\sigma}}$ $\qquad E[Y] = 0 \qquad \operatorname{Var} Y = 1$

Y è la v.a. centrata e ridotta associata a ~~X~~ X

Mean

$X \qquad \operatorname{Var} X \qquad \sqrt{\operatorname{Var} X}$ = deviazione standard

$$X \sim N(1,4)$$

$$P(X \le 2.6) =$$

$$Y = \frac{X-1}{2} \sim N(0,1)$$

$$= P\left(\frac{X-1}{2} \le \frac{2.6-1}{2}\right) = P(Y \le 0.8)$$

$$0.78814 = \Phi(0.8)$$

$$P(X \geq 0.386) = \qquad \Phi(x) = P(\underset{\substack{\uparrow \\ N(0,1)}}{Y} \leq x)$$

$$= 1 - P(X < 0.386) =$$

$$= 1 - P(X \leq 0.386) = \qquad 1 - \Phi(-0.31)$$

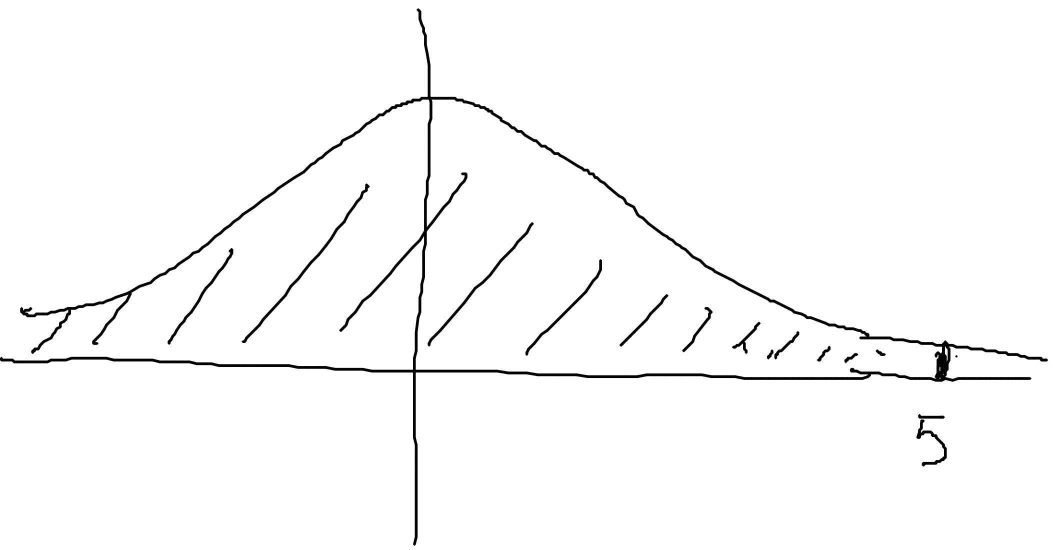
$$= 1 - P\left(\frac{X-1}{2} \leq \frac{0.386-1}{2}\right) = 1 - P(Y \leq -0.31)$$

$$= 1 - P\left(Y \leq -\frac{0.614}{2}\right) = 1 - P(Y \leq -0.307)$$

$$= \cancel{1 - P(Y \leq 0,}$$

$$= 1 - \Phi(-0.31) = 1 - (1 - \Phi(0.31)) =$$

$$\Phi(-x) = 1 - \Phi(x)$$



$$= \Phi(0.31)$$

$$= 0.62172$$

$$\Phi(-4.9)$$

Teorema. Siano $X \sim N(\mu_1, \sigma_1^2)$

$Y \sim N(\mu_2, \sigma_2^2)$

X e Y indipendenti

Allora la v. a. $Z = X+Y \sim N(\mu_1+\mu_2, \sigma_1^2+\sigma_2^2)$

Es. Siano X e Y 2 v. a. indipendenti, entrambe di legge $N(1,1)$

$Z = aX + bY$ a, b costanti reali

(a) Calc. a e b in modo che $Z \sim N(0,1)$

(b) Per $a = b = 1$, calc. $P(|Z| > 1)$

(c) Calc. a e b in modo che

$$E[Z] = 2 \quad \text{e} \quad P(Z > 1) = 0.55$$

$aX + b$

(a) $Z = (aX) + (bY) =$

$aX \sim N(a \cdot 1, a^2 1) = N(a, a^2)$

$bY \sim N(b, b^2)$

$(aX) + (bY) \sim N(a+b, a^2+b^2)$

indep

$a \begin{cases} a+b=0 \\ a^2+b^2=1 \end{cases}$

$a=-b$

$a^2 = \frac{1}{2}$

$\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

$\cancel{a = \frac{1}{\sqrt{2}}, b=}$

(b) $a = b = 1$ $\qquad Z \sim N(2,2)$

$$P(|Z|>1) = P(\{Z>1\} \cup \{Z<-1\}) =$$

$$= P(Z>1) + P(Z<-1) =$$

$$= 1 - P(Z \leq 1) + P(Z \leq -1) =$$

$$= 1 - P\left(\underbrace{\frac{Z-2}{\sqrt{2}}}_{\sim N(0,1)} \leq \frac{1-2}{\sqrt{2}}\right) + P\left(\frac{Z-2}{\sqrt{2}} \leq \frac{-1-2}{\sqrt{2}}\right)$$

$$= 1 - \phi\left(-\frac{1}{\sqrt{2}}\right) + \phi\left(-\frac{3}{\sqrt{2}}\right)$$

$$= 1 - \phi\left(-\frac{1}{\sqrt{2}}\right) + \phi\left(-\frac{3}{\sqrt{2}}\right) =$$

$$= \phi\left(\frac{1}{\sqrt{2}}\right) + 1 - \phi\left(\frac{3}{\sqrt{2}}\right) =$$

$$= \phi(0.71) + 1 - \phi(2.12) =$$

$$= 0.76115 + 1 - 0.983 =$$

$$= 0.778$$

(c) $E[Z] = 2$ $\qquad$ $a+b=2$

$P(Z>1) = 0.55$

$Z \sim N(a+b, a^2+b^2)$

$E[Z] = a+b$

$$P(Z>1) = 1 - P\Big(\underbrace{\frac{Z-(a+b)}{\sqrt{a^2+b^2}}}_{\sim N(0,1)} \le \frac{1-\overset{=2}{(a+b)}}{\sqrt{a^2+b^2}}\Big) =$$

$$= 1 - \Phi\left(\frac{-1}{\sqrt{a^2+b^2}}\right) = 0.55$$

$$a+b=2$$

$$\begin{cases} a = 6.34 \\ b = -4.34 \end{cases} \quad \underbrace{1 - \phi\left(\frac{-1}{\sqrt{a^2+b^2}}\right)}\ = 0.55$$

$$\begin{cases} \phi\left(\dfrac{1}{\sqrt{a^2+b^2}}\right) = 0.55 \\ a+b = 2 \end{cases} \qquad \begin{cases} a+b=2 \\ a^2+b^2 = 59.17 \end{cases}$$

$$\frac{1}{\sqrt{a^2+b^2}} = 0.13 \qquad a^2+b^2 = \left(\frac{1}{0.13}\right)^2 = 59.17$$

Es. Sia $X \sim N(0,1)$

Calc. la legge di $Y = X^2$

se $t \geq 0$

$$F_Y(t) = P(Y \leq t) = P(X^2 \leq t) =$$

$$= P(-\sqrt{t} \leq X \leq \sqrt{t}) = \dots$$

Se $t < 0$

$$P(X^2 \le t) = 0$$

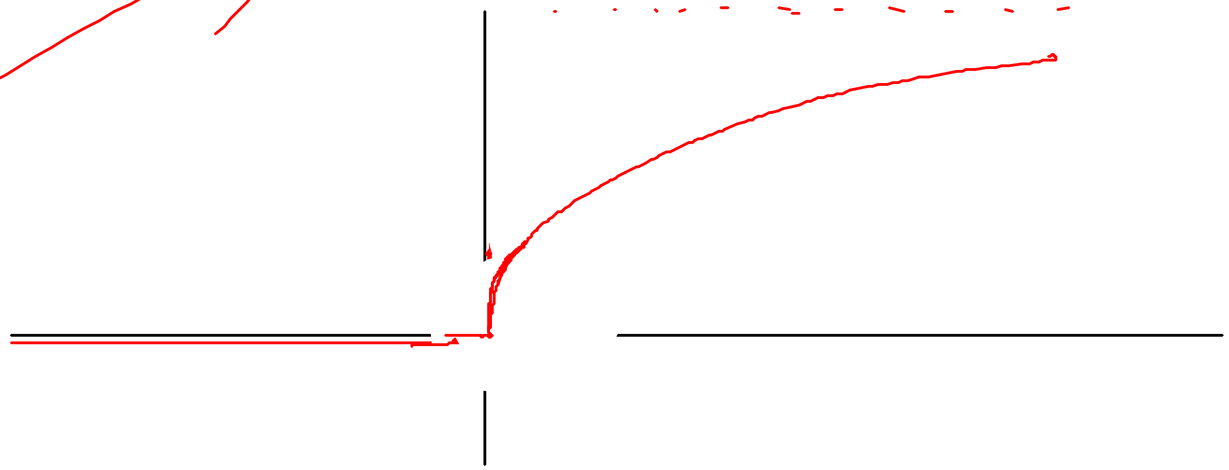
$t \geq 0$

$$P(-\sqrt{t} \le X \le \sqrt{t}) = \Phi(\sqrt{t}) - \Phi(-\sqrt{t})$$

$$P(a \le X \le b) = F(b) - F(a)$$

X continua

$$F_Y(t) = \begin{cases} 0 & t < 0 \\ \Phi(\sqrt{t}) - \Phi(-\sqrt{t}) & t \geq 0 \end{cases}$$



$$\lim_{t \to +\infty} F_Y(t) = 1 \; ?$$

$$g(t) = \begin{cases} 0 & t \leq 0 \\ \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{t}} e^{-t/2} & t > 0 \end{cases}$$

$$\phi(\sqrt{t}) - \phi(-\sqrt{t}) = 2\phi(\sqrt{t}) - 1$$

$$g(t) = 2\phi'(\sqrt{t}) \frac{1}{2\sqrt{t}} = \frac{1}{\sqrt{t}} \frac{1}{\sqrt{2\pi}} e^{-\frac{(\sqrt{t})^2}{2}} = \frac{1}{\sqrt{2\pi t}} e^{-t/2}$$

$$g(t) = \begin{cases} 0 & t \le 0 \\ \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{t}} e^{-t/2} & t > 0 \end{cases}$$

$$\frac{\left(\frac{1}{2}\right)^{1/2}}{\Gamma\left(\frac{1}{2}\right)} = \frac{1}{\sqrt{2\pi}}$$

$$\Gamma\left(\frac{1}{2}, \frac{1}{2}\right)$$

$$f(t) = \begin{cases} 0 & t \le 0 \\ \frac{\lambda^{\alpha}}{\Gamma(\alpha)} t^{\alpha-1} e^{-\lambda t} & t > 0 \end{cases}$$

$$\alpha - 1 = -\frac{1}{2}$$

$$\lambda = \frac{1}{2}$$

$$\alpha = \frac{1}{2}$$

$$\int_{\mathbb{R}} c_1 \, h(t) \, dt = 1 \implies c_1 = \frac{1}{\int h(t)\, dt}$$

$$\int_{\mathbb{R}} c_2 \, h(t) = 1 \implies c_2 = \frac{1}{\int h(t)\, dt}$$

$$\int \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \, dx = 1$$

Es.

$$\left.\begin{array}{l} X \sim N(0,1) \\ Y = \begin{cases} -1 & \frac{1}{2} \\ 1 & \frac{1}{2} \end{cases} \end{array}\right\} \text{ indip.}$$

$$Z = X \cdot Y$$

(a) Calc. la legge di Z

(b) $X + Z$ è continua?

(c) X e Z sono indipendenti? (d) Calc. $\mathrm{cov}(X,Z)$

(a) $P(XY \leq t) =$

$= P(XY \leq t, Y = 1) + P(XY \leq t, Y = -1)$

$= P(X \leq t, Y = 1) + P(-X \leq t, Y = -1)$

$P(X \in A, Y \in B) = P(X \in A) P(Y \in B)$

$A = (-\infty, t]$ $B = \{1\}$

$= P(X \leq t) \cdot P(Y = 1) + P(-X \leq t) \cdot P(Y = -1)$

$$= \frac{1}{2} P(X \leq t) + \frac{1}{2} \underline{P(-X \leq t)}$$

$$= \frac{1}{2} \phi(t) + \frac{1}{2} \phi(t) = \phi(t)$$

$$P(Z \leq t) = \phi(t)$$

quindi $Z \sim N(0,1)$

(b) $\quad X+Z = X + XY = X(Y+1)$

$$Y+1 = \begin{cases} 0 & 1/2 \\ 2 & 1/2 \end{cases}$$

$0 = P(X+Z=0) = P(\{X=0\} \cup \{Y+1=0\})$

$\{X+Z=0\} = \{X=0\} \cup \{Y+1=0\} \supseteq \{Y+1=0\}$

$P(X+Z=0) \geq P(Y+1=0) = \frac{1}{2}$

$$X \sim N(0,1)$$

$$Z \sim N(0,1)$$

$$Z = XY$$

$X+Z$ non è continua

$$X+Z \sim \underline{N(0,2)}$$

non ~~[illegible]~~ sono indipendenti

d) $\operatorname{Cov}(X, Z) = E[X \underbrace{Z}_{XY}] - E[X]\,E[\underbrace{Z}_{XY}]$

$= E[X^2 Y] - E[X]\,E[XY] =$

$= E[X^2] \cdot E[Y] - E[X] \cdot E[X] \cdot E[Y] =$

$= E[Y] \left(E[X^2] - E^2[X] \right) =$

$= \underbrace{E[Y]}_{>0} \cdot \underbrace{\operatorname{Var} X}_{1}$